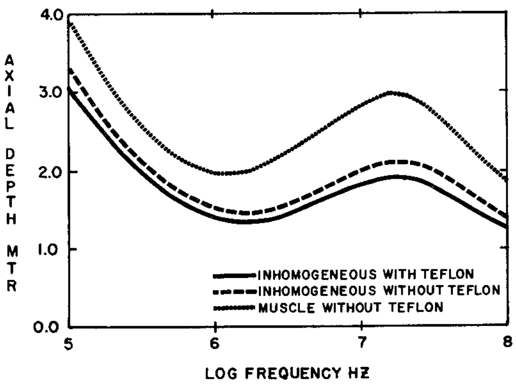

Fig 2 Axial depth as a function of frequency for a sheath helix having a lossy dielectric cylinder on its axis.

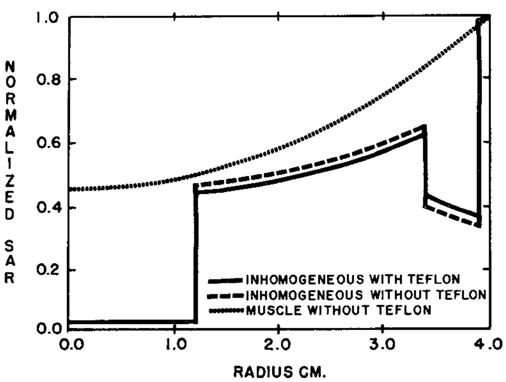

Fig. 3. Normalized SAR as a function of radius in a lossy dielectric cylinder within a sheath helix at 27.12 MHz.

Fig. 3 shows the specific absorption rate (SAR), normalized to the maximum which occurs at the skin surface, as a function of radius for a frequency of 27.12 MHz. The large decrease in energy deposition within the bone and fat layers is largely due to the decreased conductivity of those dielectrics. The fractional change in normalized SAR is more pronounced at the muscle–bone interface than at the muscle–fat or fat–skin interfaces. This is attributed to the fact that the radial component of the electric field is not negligible at the larger radii, so boundary conditions require the magnitude of the electric field to be somewhat greater in the fat layer than in the nearby regions of muscle or skin. The slight increase of deposition in the fat layer and decrease of deposition in the skin layer caused by the teflon would also be seen with a decrease in helix radius, which is consistent with the effect of teflon on axial depth.

The deep, relatively uniform, deposition of energy illustrated in Fig. 3 is in qualitative agreement with experimental results obtained using models with a thermographic camera [3] and confirms that the helical coil shows promise for use as an applicator in hyperthermia.

V. Conclusion

The numerical method described in this paper has been found to be useful for evaluation of the fields of a sheath helix in a coaxially layered lossy dielectric medium. The examples presented pertain to clinical applications and support experimental results suggesting suitability of the helical coil as an applicator in hyperthermia.

Acknowledgment

The author is greatful to Dr. R. S. Chadwick of NIH and Dr. F. W. J. Olver of NBS for valuable suggestions regarding the computation of Bessel functions having complex arguments. The assistance of R. O. Creecy and Dr. R. L. Levin with the computer facilities at NIH is also appreciated.

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Impedance Calculation of Three Narrow Resonant Strips on the Transverse Plane of a Rectangular Waveguide

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***Abstract* —A theoretical analysis has been developed to calculate the impedance of two inductive strips and one capacitive strip located on the transverse plane of a rectangular waveguide. The current ratios among the strips were determined by a variational method and then used for impedance calculations. The results can be applied to the impedance calculations of a single capacitive strip, two inductive strips, or three inductive strips as special cases.**

Manuscript received April 26, 1983; revised August 3, 1983.
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0018-9480/84/0100-0126$01.00

I. Introduction

Various discontinuities of strips or diaphragms in rectangular waveguide have been analyzed in the past. These include a single inductive strip [1], a single capacitive strip [2], an axial inductive strip [3]–[5], an axial capacitive strip [5], two inductive strips [6]–[8], three inductive strips [9]–[10], and two resonant strips [11]. The results for strips can also be applied to round posts by using a post diameter-to-strip width equivalence in the calculation [12], [13]. The effect of phase variation of the field across the post can be accounted for by a lumped element [14], [15].

This paper reports an analysis on three narrow resonant strips located on the transverse plane of a rectangular waveguide. As shown in Fig. 1, the configuration consists of two symmetrically located inductive strips with a tuning capacitive strip placed between them. This arrangement has many applications in filter and matching network designs. In many cases, the tuning capacitive strip is used for fine adjustment to compensate the design discrepancy of the inductive window. Although this structure has been commonly used, no theoretical analysis has been published. The purpose of this paper is to formulate a closed-form solution for this problem.

The resultant theoretical expression is quite general; special cases of the results lead to the previously reported solutions of a single capacitive strip, two inductive strips, and three inductive strips.

II. Theoretical Derivation

The structure being analyzed is shown in Fig. 1. The strips are assumed to be infinitesimally thin and made of perfectly conducting material. The two inductive strips of width w_1 are placed symmetrically in a rectangular waveguide, at the plane $z = 0$. The capacitive strip of width w_2 and depth d is located anywhere between the two inductive strips. The incident dominant-mode electric field is given by

$$E_t = \sin\left(\frac{\pi x}{a}\right)\exp(-\Gamma_1 z)\,\hat{y}.$$

The total normalized shunt susceptance $\bar{B}_T$ may be expressed in the following variational form [2]:

$$j\bar{B}_T = \frac{2\left[\int_S J_y(x,y)\sin\frac{\pi x}{a}\,dx\,dy\right]^2}{\frac{\Gamma_{10}}{k_0^2}\left[\left(\sum_{n=2}^{\infty}\sum_{m=0}^{\infty}+\sum_{m=1,\,\text{at}\,n=1}^{\infty}\right)\frac{(2-\delta_m)(k_0^2-m^2\pi^2/b^2)}{\Gamma_{nm}}\int_S\int_S J_y(x,y)J_y(x',y')\sin\frac{n\pi x}{a}\sin\frac{n\pi x'}{a}\cos\frac{m\pi y}{b}\cos\frac{m\pi y'}{b}\,dx\,dx'\,dy\,dy'\right]} \tag{1}$$

where

$$\delta_m = \begin{cases}1, & \text{when } m = 0\\ 0, & \text{when } m \neq 0\end{cases}$$

$$\Gamma_{nm} = \left(\frac{m^2\pi^2}{b^2}+\frac{n^2\pi^2}{a^2}-k_0^2\right)^{1/2}$$

$$S = S_a + S_b + S_c$$

$$k_0 = \frac{2\pi}{\lambda}$$

and $J_y(x,y)$ is the y-directed current density in the strips of surface S. Using a method similar to that of Lewin [16], the expression of (1) can be shown to be stationary for small variations in J_y about its correct value. Use of an approximate form for J_y in (1) yields a lower bound on the true value of the susceptance.

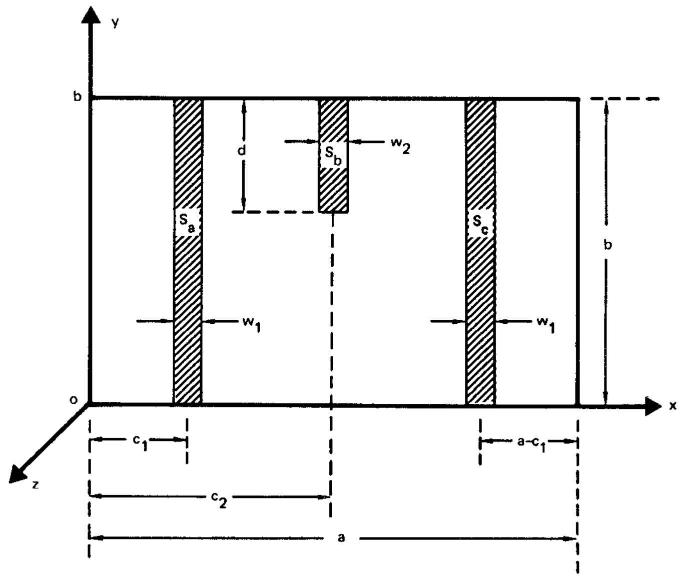

Fig. 1. Cross section of a rectangular waveguide with three thin strips in the same transverse plane.

For narrow strips, the current density on each strip can be assumed as constant along the X-direction. It can be written in the form

$$\begin{aligned}J_y(x,y) &= A\left[u\left(x-c_1+\frac{w_1}{2}\right)-u\left(x-c_1-\frac{w_1}{2}\right)\right]\\ &\quad+fA\sin k(y-b+d)\left[u\left(x-c_2+\frac{w_2}{2}\right)-u\left(x-c_2-\frac{w_2}{2}\right)\right]\\ &\quad+A\left[u\left(x-a+c_1+\frac{w_1}{2}\right)-u\left(x-a+c_1-\frac{w_1}{2}\right)\right]\end{aligned} \tag{2}$$

where A is an amplitude constant, f is the current ratio, and $u(x)$ is the unit step function. k is defined as

$$k = \frac{\pi}{2d}. \tag{3}$$

The reasons for this definition can be found in [2], and therefore will not be repeated here.

Substituting the current density from (2) into (1), we obtain

$$j\bar{B}_T = \frac{2\frac{a^2}{\pi^2}\{F_1+fF_2\}^2}{\frac{\Gamma_{10}}{k_0^2}\left\{\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{2\left(k_0^2-\frac{m^2\pi^2}{b^2}\right)}{\Gamma_{nm}}f^2P_{nm}^2+\sum_{n=2}^{\infty}\frac{k_0^2}{\Gamma_{n0}}[Q_n+fS_n]^2\right\}} \tag{4}$$

where

$$P_{nm} = \frac{a}{2n\pi}\left[\cos\frac{n\pi\left(c_2+\frac{w_2}{2}\right)}{a} - \cos\frac{n\pi\left(c_2-\frac{w_2}{2}\right)}{a}\right]$$

$$\cdot\left\{\frac{1}{k+\frac{m\pi}{b}}\left[\cos\frac{m\pi(b-d)}{b} - \cos(m\pi+kd)\right]\right.$$

$$\left.+\frac{1}{k-\frac{m\pi}{b}}\left[\cos\frac{m\pi(b-d)}{b} - \cos(m\pi-kd)\right]\right\}$$

$$Q_n = \frac{ab}{n\pi}\left[\cos\frac{n\pi\left(c_1+\frac{w_1}{2}\right)}{a} - \cos\frac{n\pi\left(c_1-\frac{w_1}{2}\right)}{a}\right.$$

$$\left.+\cos\frac{n\pi\left(a+c_1+\frac{w_1}{2}\right)}{a} - \cos\frac{n\pi\left(a-c_1-\frac{w_1}{2}\right)}{a}\right]$$

$$S_n = \frac{2ad}{n\pi^2}\left[\cos\frac{n\pi\left(c_2+\frac{w_2}{2}\right)}{a} - \cos\frac{n\pi\left(c_2-\frac{w_2}{2}\right)}{a}\right]$$

$$F_1 = b\left[\cos\frac{\pi\left(c_1-\frac{w_1}{2}\right)}{a} - \cos\frac{\pi\left(c_1+\frac{w_1}{2}\right)}{a}\right.$$

$$\left.+\cos\frac{\pi\left(a-c_1-\frac{w_1}{2}\right)}{a} - \cos\frac{\pi\left(a-c_1+\frac{w_1}{2}\right)}{a}\right]$$

$$F_2 = \frac{2d}{\pi}\left[\cos\frac{\pi\left(c_2-\frac{w_2}{2}\right)}{a} - \cos\frac{\pi\left(c_2+\frac{w_2}{2}\right)}{a}\right].$$

The only unknown in (4) is the current ratio f, which can be determined by extremizing the variational expression for $j\bar{B}_T$. Putting $d\bar{B}_T/df = 0$ yields

$$A_1 f^2 + A_2 f + A_3 = 0 \tag{5}$$

where

$$A_1 = 2\left[F_1F_2\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}U_{nm} + F_1F_2\sum_{n=2}^{\infty}V_nS_n^2 - F_2^2\sum_{n=2}^{\infty}V_nQ_nS_n\right]$$

$$A_2 = 2\left[F_1^2\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}U_{nm} + F_1^2\sum_{n=2}^{\infty}V_nS_n^2 - F_2^2\sum_{n=2}^{\infty}V_nQ_n^2\right]$$

$$A_3 = 2F_1^2\sum_{n=2}^{\infty}V_nQ_nS_n - 2F_1F_2\sum_{n=2}^{\infty}V_nQ_n^2$$

and

$$U_{nm} = 2\frac{a^2}{\pi^2}\frac{\Gamma_{10}}{k_0^2}\frac{2\left(k_0^2-\frac{m^2\pi^2}{b^2}\right)}{\Gamma_{nm}}P_{nm}^2$$

$$V_n = 2\frac{a^2}{\pi^2}\frac{\Gamma_{10}}{\Gamma_{n0}}.$$

The solutions to (5) are

$$f = \frac{-A_2 \pm \sqrt{A_2^2 - 4A_1A_3}}{2A_1}. \tag{6}$$

One of the solutions results in $\bar{B}_T = 0$ and should be rejected.

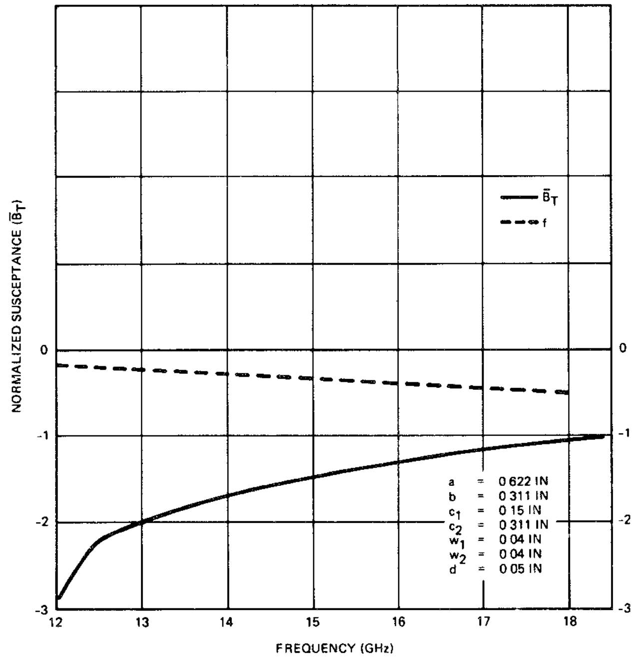

Fig. 2. Normalized susceptance and current ratio as a function of frequency for strip depth $d = 0.05$ in.

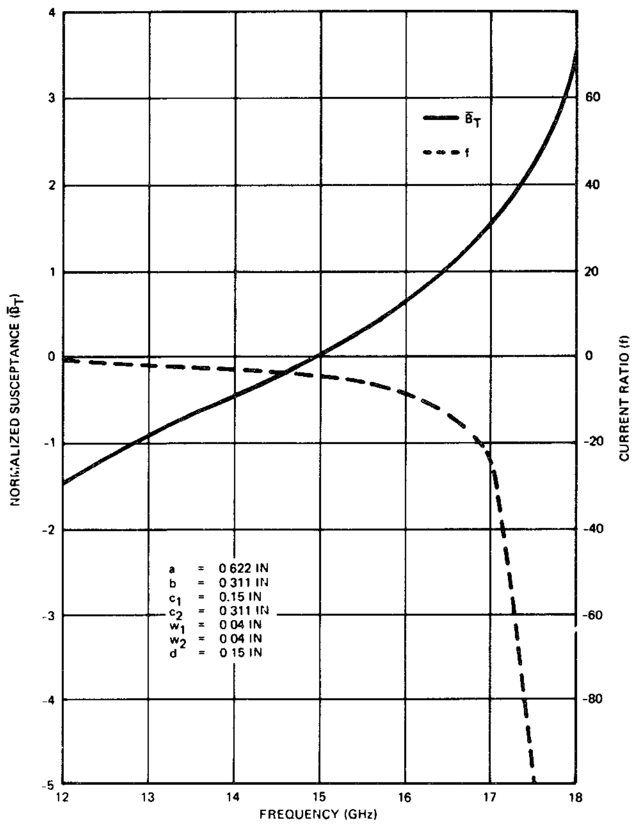

Fig. 3. Normalized susceptance and current ratio as a function of frequency for strip depth $d = 0.15$ in.

III. Theoretical Discussion

A computer program has been developed based on this analysis. Using a *Ku*-band waveguide ($a = 0.622$ in, $b = 0.311$ in) as an example, the theoretical results are plotted in Figs. 2, 3, and 4 as a function of frequency for three different depths of the capacitive strip. As the capacitive strip is barely inserted into the waveguide, the impedance is dominated by the two inductive

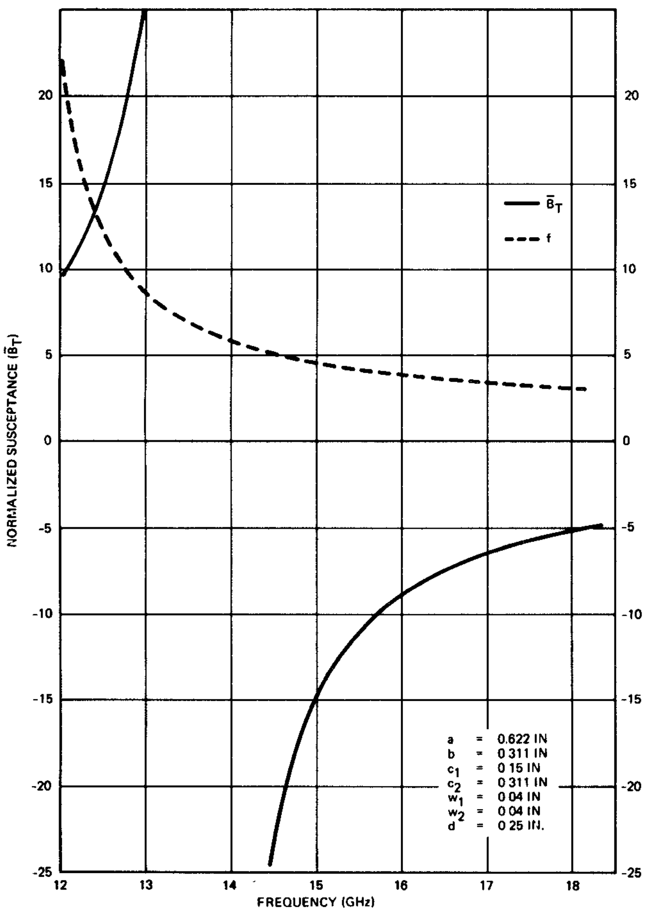

Fig. 4. Normalized susceptance and current ratio as a function of frequency for strip depth $d = 0.25$ in.

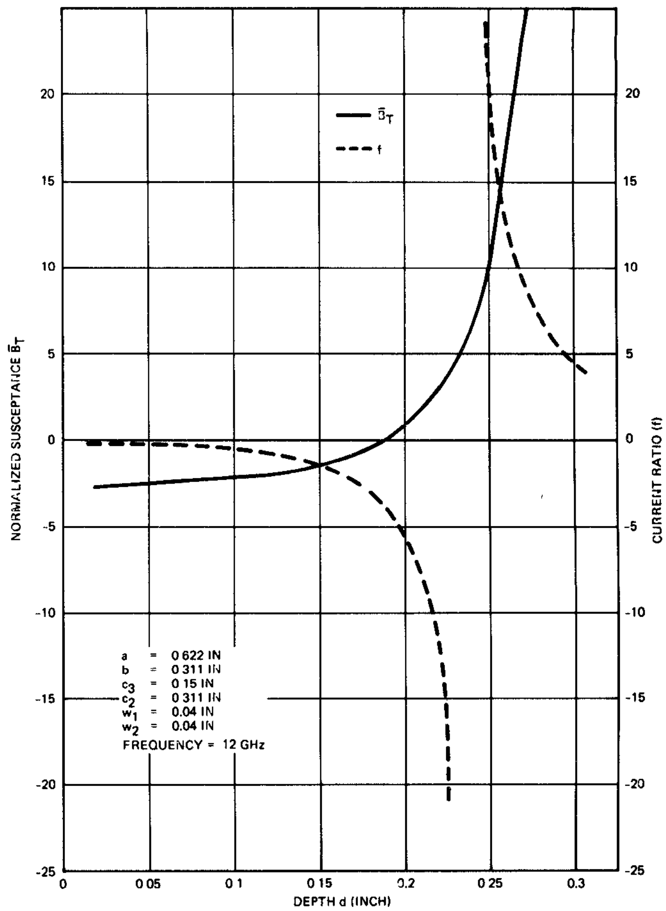

Fig 5. Normalized susceptance and current ratio as a function of strip depth at 12 GHz.

strips and the resultant total susceptance is similar to that of two inductive strips [6], as shown in Fig. 2. As the depth of the capacitive strip increases, the capacitance becomes larger, and at certain frequencies it cancels with the inductance. As shown in Fig. 3, the total normalized susceptance is zero at 15 GHz and the waveguide is transparent at this frequency. A further increase of the capacitive strip depth results in strong interactions among strips, and a very high susceptance over the entire frequency range as shown in Fig. 4.

Experimental measurements on the reflection coefficient have confirmed the above theoretical predictions. For $d = 0.15$ in, a resonance was observed at frequencies near 15 GHz with very low reflection. For $d = 0.25$ in, it was found that the reflection is very high and the circuit acts like a short circuit.

For a fixed frequency, the total normalized susceptance can be tuned to zero by varying the depth of the capacitive strip. The results are shown in Figs. 5 and 6.

IV. Special Cases

The results given by (4) can be used for the special cases shown in Fig. 7. They are a single capacitive strip, two inductive strips, and three inductive strips.

A. Single Capacitive Strip

In this case, the following parameters are set to zero:

$$w_1 = 0$$

$$Q_n = 0$$

$$F_1 = 0.$$

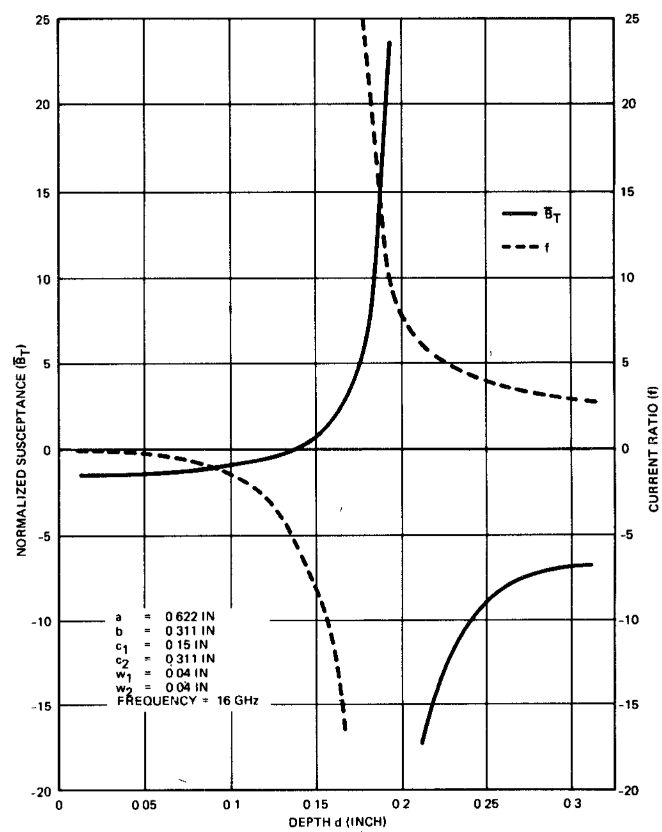

Fig. 6. Normalized susceptance and current ratio as a function of strip depth at 16 GHz.

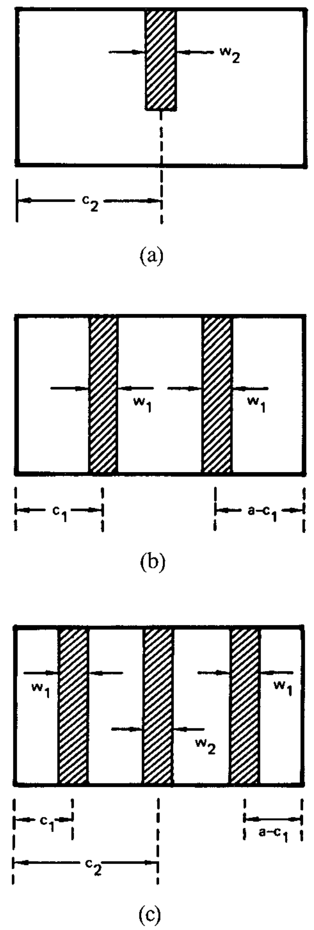

Fig. 7. Three special cases. (a) A single capacitive strip. (b) Two symmetrical inductive strips. (c) Three inductive strips.

Equation (4) reduces to

$$j\bar{B}_T = \frac{2\frac{a^2}{\pi^2}F_2^2}{\frac{\Gamma_{10}}{k_0^2}\left\{\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{2\left(k_0^2-\frac{m^2\pi^2}{b^2}\right)}{\Gamma_{nm}}+\sum_{n=2}^{\infty}\frac{k_0^2}{\Gamma_{n0}}S_n^2\right\}}. \quad (7)$$

This expression is equivalent to the results previously reported in [2, eq. (13)].

B. Two Symmetrical Inductive Strips

In this case, the following parameters are zero:

$$w_2 = 0$$
$$P_{nm} = 0$$
$$S_n = 0$$
$$F_2 = 0.$$

Equation (4) reduces to

$$j\bar{B}_T = \frac{2\frac{a^2}{\pi^2}F_1^2}{\frac{\Gamma_{10}}{k_0^2}\left\{\sum_{n=2}^{\infty}\frac{k_0^2}{\Gamma_{n0}}Q_n^2\right\}}. \quad (8)$$

This is equivalent to [6, eq. (6)] with $f = 1$ in [6] for two symmetrical inductive strips.

C. Three Inductive Strips

In this case, the following condition is set:

$$P_{nm} = 0.$$

Equation (4) reduces to

$$J\bar{B}_T = \frac{2\frac{a^2}{\pi^2}\{F_1 + fF_2\}^2}{\Gamma_{10}\sum_{n=2}^{\infty}\frac{1}{\Gamma_{n0}}[Q_n + fS_n]^2}. \quad (9)$$

This is equivalent to [9, eq. (4)] if f_1 and f_2 of [9] are set to be

$$f_1 = \frac{\pi b}{2d}f$$
$$f_2 = 1.$$

The first condition is arbitrary because f or f_1 is solved by extremizing $j\bar{B}_T$. The second condition is due to the geometrical symmetry of the two other strips.

V. Conclusions

A theoretical analysis has been derived for three narrow resonant strips in a rectangular waveguide. As special cases, the resultant closed-form expression can be used to calculate a single capacitive strip, two symmetrical inductive strips, and three inductive strips. The results should be useful for filter and matching circuit designs.

VI. Acknowledgement

The author would like to thank Dr. M. Chen of the TRW Antenna Laboratory for providing the experimental results.

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